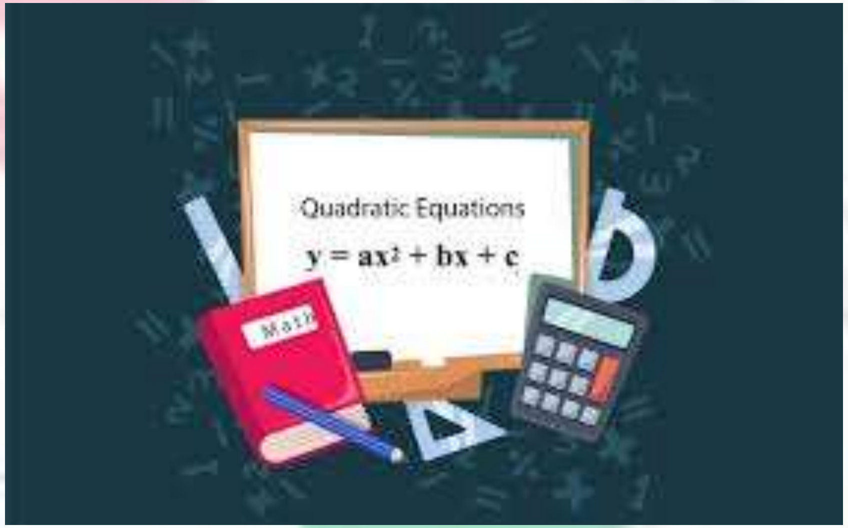
Quadratic Equation



SOLVING QUADRATIC EQUATION USING DELTA

Recall Quadratic Equation

$ax^2 + bx + c = 0$

Discriminant $\Delta = b^2 - 4ac$

Application: Find the discriminant

$$2x^2 - 3x + 1 = 0$$

a=2

b=-3

c=1

$$8x^2 + 2x = 0$$

a=8

b=2

c=0

$$-7x^2 + 3x - 2 = 0$$

a=-7

b=3

c=-2

Application: Find the discriminant

$$\Delta = b^2 - 4ac$$

$$2x^2 - 3x + 1 = 0$$

a=2

b=-3

c=1

$$\Delta = (-3)^2 - 4(2)(1) = 1$$

$$8x^2 + 2x = 0$$

a=8

b=2

c=0

$$\Delta = (2)^2 - 4(8)(0) = 4$$

$$-7x^2 + 3x - 2 = 0$$

a=-7

b=3

c=-2

$$\Delta = (3)^2 - 4(-7)(-2) = -47$$

How does Δ help us to solve quadratic equation ?

Signs of Δ

Positive

Negative

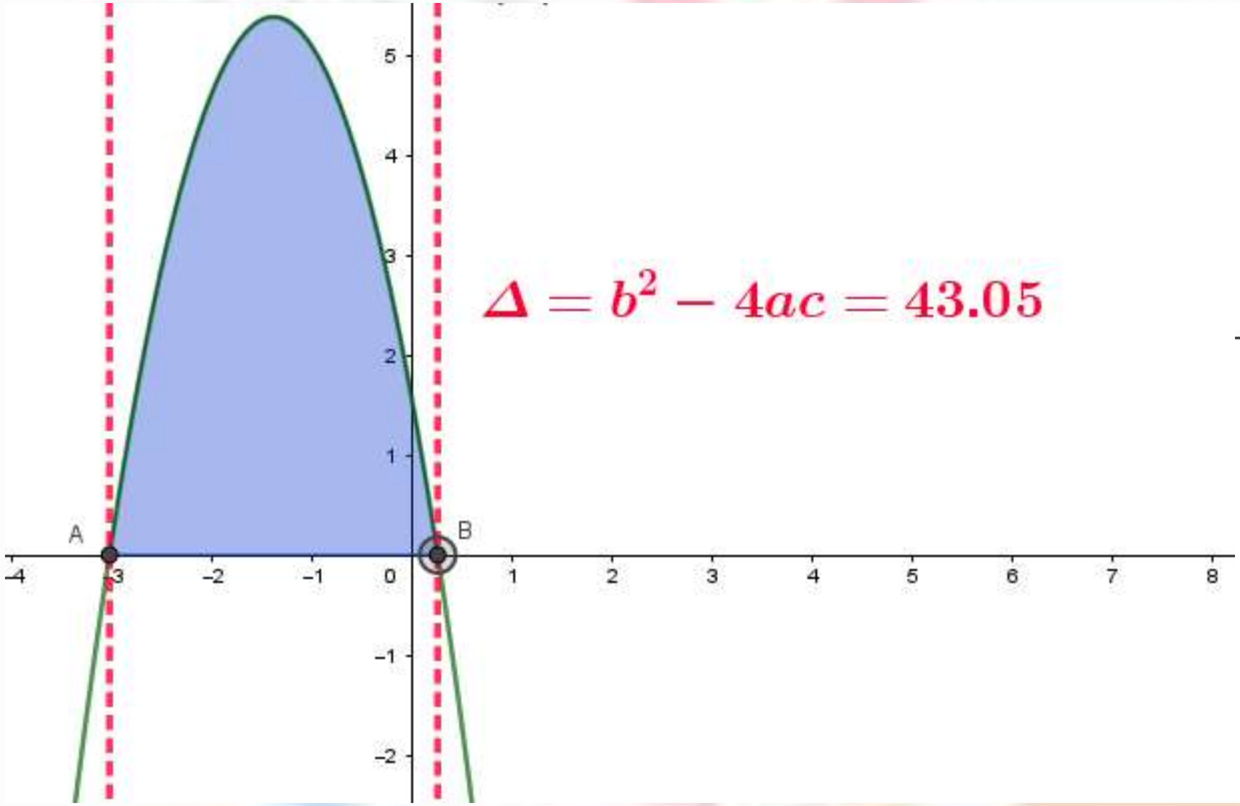
Zero

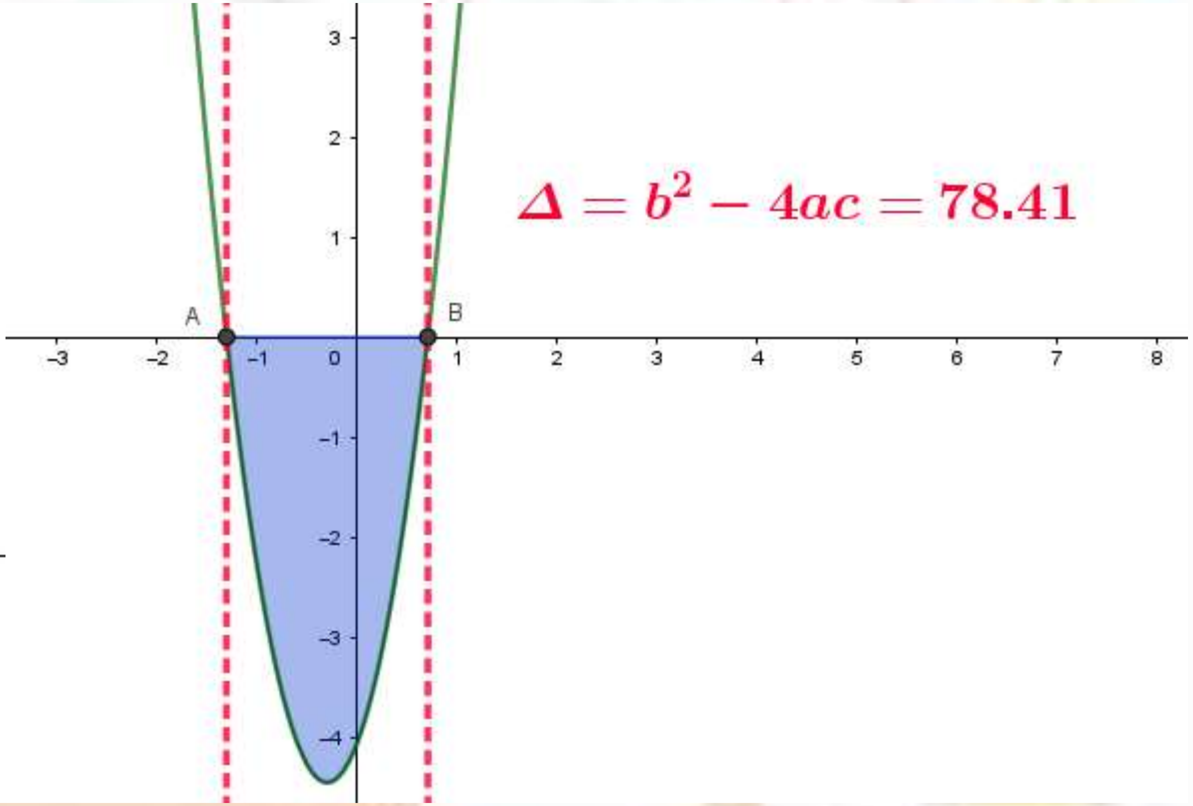
Case 1: If delta is positive, there're 2 distinct roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Discriminant

Case 1: If delta is positive, there're 2 distinct roots

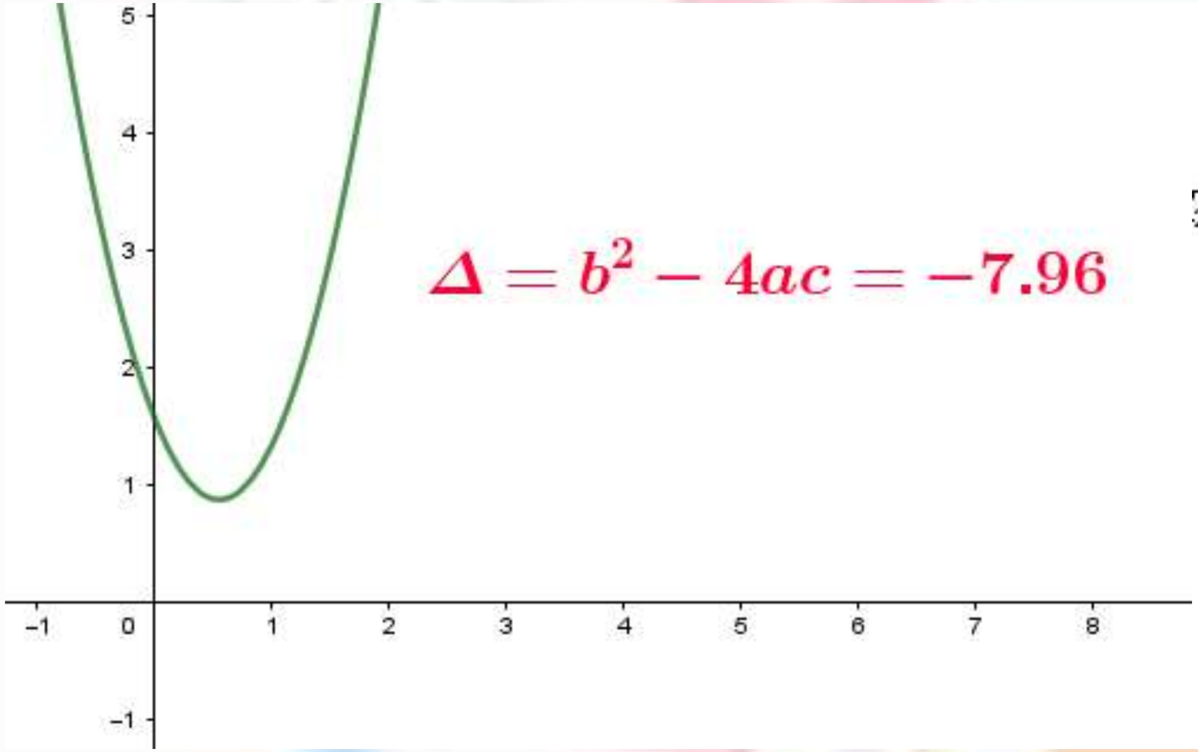


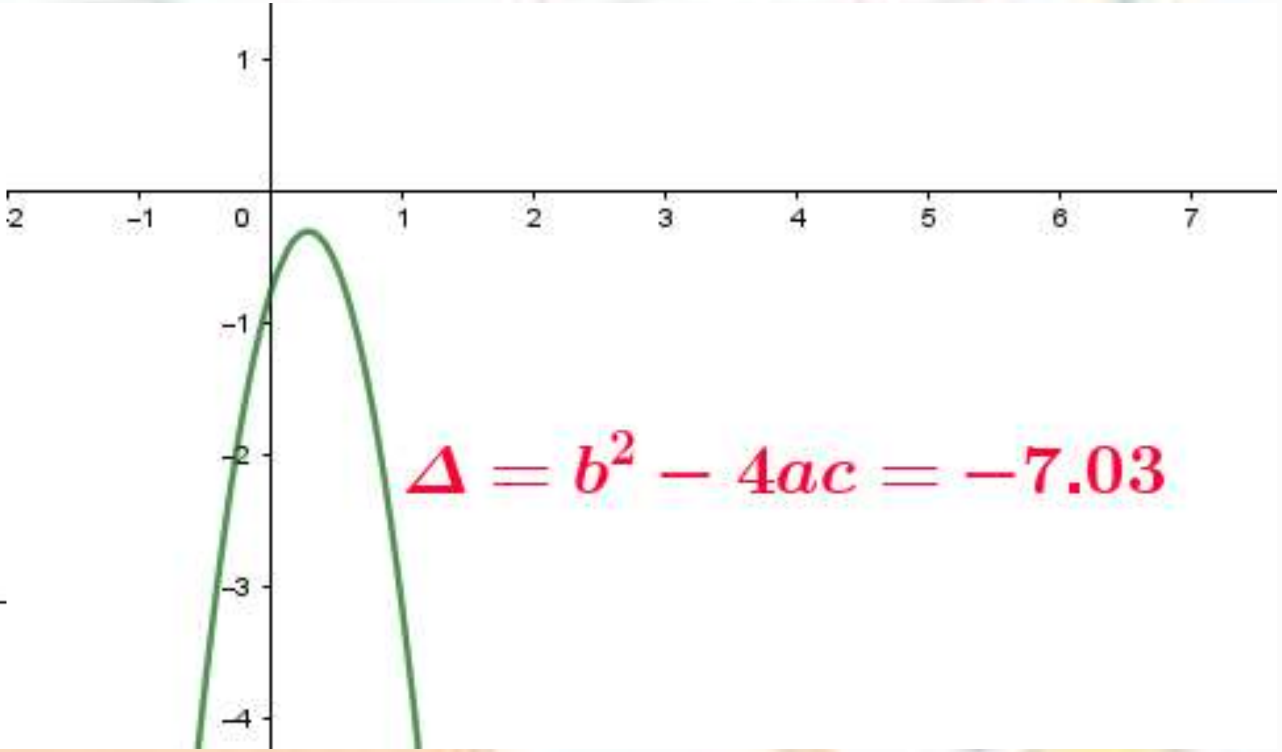


Case 2: If delta is negative

NO Real Roots

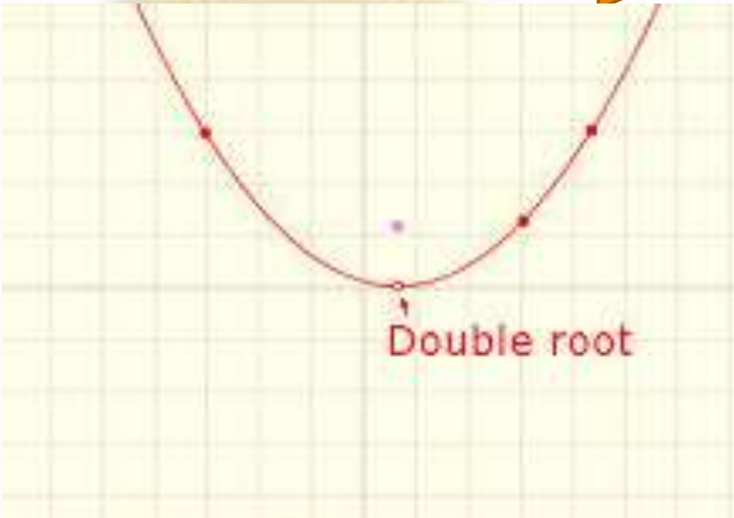
Case 2: If delta is negative

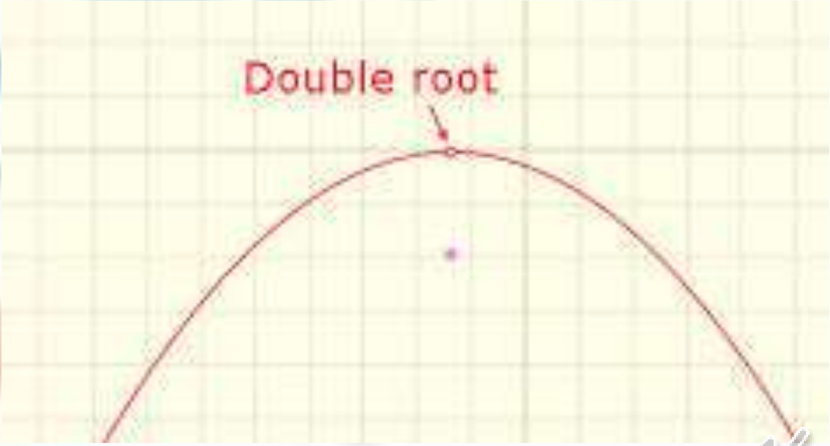




Case 3 : If delta is zero , there's a Double Root

$$x' = x'' = \frac{-b}{2a}$$





Reduced Discriminant Δ'

When b is even number
b=2b'

$$\Delta' = (b')^2 - ac$$

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$$b' = \frac{b}{2}$$

- If $\Delta' < 0$, $\quad$ *then No Real Roots*
- If $\Delta' = 0$, $\quad$ *then we've double root* $x' = x'' = \frac{-b}{a}$
- If $\Delta' > 0$, $\quad$ *then we've two distinct roots*

$$x' = \frac{-b' + \sqrt{\Delta'}}{a} \quad \& \quad x'' = \frac{-b' - \sqrt{\Delta'}}{a}$$

Solved Exercises :

1) $2x^2 - 3x + 5 = 0$

$\Delta = (-3)^2 - 4(2)(5) = 9 - 40 = -31 < 0$

NO Real Roots

Solved Exercises :

2) $3x^2 - 2x - 1 = 0$

$\Delta = (-2)^2 - 4(3)(-1) = 4 + 12 = 16 > 0$,

$$\boldsymbol{then\ we've\ two\ distinct\ roots}$$

$$x' = \frac{-b+\sqrt{\Delta}}{2a} = \frac{2+\sqrt{16}}{2(3)} = 1 \qquad \&$$

$$x'' = \frac{-b-\sqrt{\Delta}}{2a} = \frac{2-\sqrt{16}}{2(3)} = \frac{-1}{3}$$

Solved Exercises :

2) $3x^2 - 2x - 1 = 0$

$\Delta' = (-1)^2 - (3)(-1) = 1 + 3 = 4 > 0$,

$\boldsymbol{then\ we've\ two\ distinct\ roots}$

$$x' = \frac{-b' + \sqrt{\Delta'}}{a} = \frac{1 + \sqrt{4}}{(3)} = 1 \qquad \&$$

$$x'' = \frac{-b' - \sqrt{\Delta'}}{a} = \frac{1 - \sqrt{4}}{(3)} = \frac{-1}{3}$$

$b' = \frac{b}{2}$

$b' = \frac{-2}{2}$

$b' = -1$

Solved Exercises :

3) $x^2 - 6x + 9 = 0$

$\Delta = (-6)^2 - 4(1)(9) = 36 - 36 = 0$

$$x' = x'' = \frac{-b}{2a}$$

Remark

If a+b+c=0 , then x'=1 and x''=$\frac{c}{a}$

Example : $2x^2 + 3x - 5 = 0$

2+3+(-5)=0 then x'=1 x''=$\frac{-5}{2}$

Remark

If a-b+c=0 , then x'=-1 and x''=$\frac{-c}{a}$

Note:
b=a+c

Example : $5x^2 - 2x - 7 = 0$

5+2-7=0 then x'=-1 x''=$\frac{7}{5}$

Time for Practice

Solve the following quadratic equations :

a) $x^2 - 8x + 12 = 0$

b) $x^2 + 5x + 7 = 0$

c) $3x^2 + 6x - 9 = 0$

d) $-8x^2 + 6x + 2 = 0$

Time to check your work

Equations	Answer
a) $x^2 - 8x + 12 = 0$	$x = 6 \quad or \quad x = 2$
b) $x^2 + 5x + 7 = 0$	No solution
c) $3x^2 + 6x - 9 = 0$	$x = -3 \quad or \; x = 1$
d) $-8x^2 - 6x + 2 = 0$	$x = -1 \; or \; x = \frac{1}{4}$